MATH ROUTINES

This program simply computes the 4 routines: SINE, COSINE, ARCTANGENT, & SQUARE ROOT. When the program is loaded, press 1 through 4 on the keypad to get the functions. You can check these functions on a calculater and see how accurate they are. Most of the answers come within 3-thousanths of a degree of the calculation. If you are programming a 3-dimensional game and using these functions for vectors & angles etc., they are more than accurate for your purposes. The SINE function is contained in the lines 100-140. The COSINE function is contained in lines 200-240. The ARCTANGENT function is in lines 300-360. The SQUARE ROOT function can be found in line 400-450. The numbers that can be used in the SINE function are 6-99. The COSINE function can use numbers 0-103. Numbers for use in the ARCTANGENT function are .001-.999 (NOTE: the decimal point is already on the screen). SQUARE ROOT uses numbers 2-32767.

To the right..... Lines 60-110 calculate the cosine of 90-S

```
10 IF S>45 GOTO 60
20 LET S=174*S/10
30 LET T=S/10
40 LET S=S-T*T/1000*T/6+T*T/1000*T/100*T/100*T/120
50 RETURN
60 LET S=90-S
70 LET S=174*S/10
80 LET T=S/10
90 LET S=1000-T*T/20+T*T/1000*T/100*T/24
100 LET S=S-T*T/1000*T/100*T/100*T/100*T/720
110 RETURN
```

Listing 1: BASIC routine for calculating the sine function. The sine is calculated in parts per thousand and the value of the sine is returned to the calling routine in variable S.

Listing 1 shows the S=sin (S) routine. S is an angle ranging from 0 to 90 degrees. The routine returns the sine of S in variable S in parts per thousand (1000 times the sin (S)). This routine makes use of the series:

$$\sin(S)=S-S^3/3!+S^5/5!-S^7/7!+\ldots$$

where S is in radians. Line 20 converts S from degrees to radians times 1000. Line 40 is the sine series in a form suitable for 16 bit integer mathematics. Beyond 45 degrees this series gave poor results. For values over 45 degrees line 10 transfers control to line 60. Lines 60 to 110 take the cosine of 90–S using the series:

$$\cos(S)=1-S^2/2!+S^4/4!-S^6/6!+\ldots$$

So there are the listings of the 4 functions, you can refer to the tape program if you prefer.

```
10 IF U>960 LET U=U+4
20 LET T=U/10
30 LET S=U-T*T/1000*T/3+T*T/1000*T/100*T/100*T/5
40 LET S=S-T*T/1000*T/100*T/100*T/100*T/100*T/7
50 LET S=S*10/174
60 RETURN
```

Listing 2: Routine for calculating the arctangent of U in parts per thousand. The result is returned to the calling routine in variable S.

Listing 2 is a similar routine to calculate the arctangent of a ratio U expressed in parts per thousand (U times 1000). The series used here is:

$$\arctan(U)=U-U^3/3+U^5/5-U^7/7+\ldots$$

It is not valid for $U \geq 1$. Line 10 is an adjustment to improve accuracy for values approaching 1. Notice that here the result S is in radians and must be converted to degrees immediately before exiting (line 50).

SQUARE ROOT is as follows:

```
400 INPUT " NUMBER?"S
410 A=S÷2
420 FOR T=1TO 10;IF A=0A=1
430 B=S÷A;IF A=BGOTO 450
440 A=(A+B)÷2;NEXT T
450 PRINT " SQUARE ROOT= ",#1,A,".",RM+3
460 GOTO 400
```

MATH ROUTINES

LISTING

PROGRAM:

```
 1.
 2:RETURN
 3.MATH ROUTINES FOR 3-D SIMULATION
 5.TIM HAYS
10 NT=1;A=0;BC=200;FC=7
20 CLEAR ;PRINT
70 PRINT "←↓→↑←↓→MATH ROUTINES←↓→↑←↓
   PRINT ;PRINT "     FUNCTION:
50 PRINT ;PRINT "     1-SINE";PRINT "
   COSINE";PRINT "     3-ARCTANGENT
51 PRINT "     4-SQUARE ROOT
55 BOX 0,0,160,88,3;BOX 0,0,156,84,3
60 IF &(23)=8A=1
70 IF &(22)=8A=2
80 IF &(21)=8A=3
82 IF &(23)=4A=4
85 IF AGOTO 95
90 GOTO 60
95 CLEAR ;PRINT ;GOTO 100*A
00 INPUT " NUMBER?"S
   IF S>45GOTO 160
   S=174*S÷10;T=S÷10
50 S=S-T*T÷1000*T÷6+T*T÷1000*T÷100*T÷100*T÷120
60 PRINT " SIN= .",#1,S
60 A=1;GOTO 800
70 S=90-S;S=174*S÷10;T=S÷10
70 S=1000-T*T÷20+T*T÷1000*T÷100*T÷24
80 S=S-T*T÷1000*T÷100*T÷100*T÷100*T÷720
90 S=S-1;GOTO 140
00 INPUT " NUMBER?"S
   S=174*S÷10;T=S÷10
10 S=1000-T*T÷20+T*T÷1000*T÷100*T÷24
20 S=S-T*T÷1000*T÷100*T÷100*T÷100*T÷720
   PRINT " COS= . ",#1,S-1
   A=2;GOTO 800
   INPUT " NUMBER? ."U
   IF U>960U=U+4
   T=U÷10
   S=U-T*T÷1000*T÷3+T*T÷1000*T÷100*T÷100*T÷5
   S=S-T*T÷1000*T÷100*T÷100*T÷100*T÷7
   S=S*10÷174
   PRINT " ARCTAN= ",#1,S,".",RM
   A=3;GOTO 800
   INPUT " NUMBER?"S
   A=S÷2
   FOR T=1TO 10;IF A=ØA=1
   B=S÷A;IF A=BGOTO 450
   A=(A+B)÷2;NEXT T
   PRINT " SQUARE ROOT= ",#1,A,".",RM+3
   A=4;GOTO 800
```

SUBROUTINE

COMMENTS:

```
                ;heading
2-
                ;border
                ;sine function
                ;cosine function
                ;arctan function
```

```
;square root function
;this version to be
. used where you need
. speed,otherwise use
        new routine
;corrects ±.3% difference
```

SUBROUTINE & NEW EXPANDED VERSION OF SQUARE ROOT

SUBROUTINE:

```
800 PRINT;PRINT " SAME
FUNCTION,(1)
810 PRINT "  MENU, (2)
820 IF &(23)=8CLEAR ;
PRINT ;GOTO 100*A
830 IF &(22)=8A=Ø;GOTO 20
840 GOTO 820
```

NEW-MORE ACCURATE SQUARE ROOT FUNCTION: replace 400-460 with the following.(takes longer than original, to the left, but is more accurate.) This addition is already in your program.

```
400 INPUT " NUMBER?"S
410 A=S÷2
420 FOR T=1TO 1Ø;IF A=ØA=1
430 B=S÷A;IF A=BGOTO 450
440 A=(A+B)÷2;NEXT T
445 B=S÷A;F=RM;E=RM
450 S=RM;C=S÷2
460 FOR T=1TO 30;IF C=ØC=1
470 D=S÷C;IF C=DGOTO 485
480 C=(C+D)÷2;NEXT T
485 IF C*1Ø+E>9ØE=E+2;
F=F+3
490 PRINT " SQUARE ROOT=",
#1,A,".",(C*1Ø+E)÷2+F
500 A=4;GOTO 8ØØ
```

0-45°

$$\sin x = x\left[1 - \frac{x^2}{6}\left(1 - \frac{x^2}{20}\right)\right]$$

SIN

45-90°

$$\sin x = \cos(90° - x)$$

0-45° =

$$\cos x = 1 + \frac{x^2}{2}\left(-1 + \frac{x^2}{12}\right)$$

COS

45°-90°

$$\cos x = \sin(90 - x)$$

$$\tan x = \frac{\sin x}{\cos x}$$

TAN

$$\tan x = \frac{x(15 - x^2)}{15 - 6x^2}$$

45-90

$$\tan x = \frac{1}{\tan(90° - x)}$$

90-135

$$\tan x = \frac{-1}{\tan(135° - x)}$$

135°-180

$$\tan x = -\tan(180 - x)$$